IDENTIFYING S-PARAMETER MODELS IN THE LAPLACE DOMAIN FOR HIGH FREQUENCY MULTIPORT LINEAR NETWORKS

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ABSTRACT

The aim is to build a model for the S-parameter matrix of a linear, time invariant multiple-port system operating at RF frequencies. The proposed method delivers stable, accurate models starting from measurements as well as from simulation data. The use of orthogonal polynomials allows high model orders.

1. INTRODUCTION

A linear or linearised high frequency n-port device is characterised by its scattering parameters (S-parameters). In general, the S-matrix is described by an irrational expression in the Laplace variable $s = j\omega$. For a limited frequency band, it can be shown that approximation by a rational model of finite order in s is possible [1]. This means the S-parameters of an n-port can be modelled using a common denominator rational model as shown below,

$$S_{ij}(\omega) = \frac{\sum_{r=0}^{z_{[i,j]}} a_{[i,j]r} \cdot (j\omega)^r}{\sum_{k=0}^{d} b_k \cdot (j\omega)^k} \qquad i, j = 1 \ldots n \quad (1)$$

where $z_{[i,j]} + d + 1 \le 2 \times F$. $z_{[i,j]}$ represents the order of the numerator corresponding with S_{ij} and d is the order of the common denominator. F denotes the number of measured frequencies. The parameters to be estimated are $a_{[i,j]r}$ and b_k.

It is not obvious to model S-parameters over a wide frequency range because the problem is often ill-conditioned. A possible solution is to divide the frequency range into narrow sub-ranges [2] but this results in models that are complex, which causes the need for robust model reduction techniques. Other disadvantages of this approach are the absence of selection rules for choosing number and position of the frequency bands, the large amount of steps required to build a model, and the limitation to single input, single output (SISO) estimation techniques (only 1 S-parameter is modelled at a time).

In this paper a more straightforward solution is used: multiple input, multiple output (MIMO) estimation techniques are used to model the whole S-matrix in one step. The problem of ill-conditioning during estimation is solved by using orthogonal Forsythe polynomials [3]. As noise is always present, a maximum likelihood estimator (MLE) is used in the frequency domain [4]. The noise properties can either be measured, or can be assumed to follow some law (e.g. white noise) when dealing with simulation data. The noise is assumed to be uncorrelated over the frequency and the correlation between the different measured S-parameters can be ignored. Because the uncertainty on the S-parameters is known, model errors can be detected. A complete description of the model construction and the numerical implementation details can be found in [5].

2. METHOD DESCRIPTION

A. MODEL STRUCTURE

Consider the case of an n-port. Each S-parameter can be written in the form (1). The common denominator model permits to include the relation between the S-parameters in the model. Thus the Scattering matrix has the following form:

$$[S(\omega)] = [N(\omega)] / D(\omega) \qquad (2)$$

Hereby, $[N(\omega)]$ is an nxn matrix of polynomials. $D(\omega)$ is a single polynomial.

$$N_{ij}(\omega) = \sum_{r=0}^{z_{[i,j]}} a_{[i,j]r} \cdot (j\omega)^r$$
$$D(\omega) = \sum_{k=0}^{d} b_k \cdot (j\omega)^k \qquad (3)$$

Assume the S-matrix $S_m(\omega_k)$ to be measured or simulated at a discrete set of F angular frequencies ω_k.The introduction of additive zero mean complex Gaussian noise $S_N(\omega_k)$ leads to the following model equation for the measured or simulated S-parameter at frequency ω_k:

$$S_{m[i,j]}(\omega_k) = S_{[i,j]}(\omega_k) + S_{N[i,j]}(\omega_k) \qquad (4)$$

where $E\{S_N(\omega_k)\} = 0$.

B. ESTIMATION METHOD

The Maximum Likelihood estimation of the parameters is obtained by minimizing the cost function $V(S_m, \theta)$ with respect to the parameters θ.

$$V(S_m, \theta) = \varepsilon(S_m, \theta)^H \varepsilon(S_m, \theta)$$

$$\varepsilon(S_m, \theta) = [\varepsilon_1^T \ldots \varepsilon_F^T]^T$$

$$\varepsilon_k = W(\omega_k)\mathrm{vec}(S_m(\omega_k)D(b, \omega_k) - N(a, \omega_k)$$

$$\theta = [a^T b^T]^T$$

The measurement points are weighted according to their variances. The weighting function $W(\omega_k)$ for the MLE is a n^2x n^2 diagonal matrix which can be written as follows:

$$W(\omega_k) = \frac{\sigma_S^{-1}(\omega_k)}{\sqrt{D(b, \omega_k)D^*(b, \omega_k)}} \qquad (5)$$

where $\sigma_S^2(\omega_k)$ is the covariance matrix of the measurements at frequency ω_k.

C. VALIDATION OF THE MODEL

A tool for validation is the value of the cost in the estimates, which can reveal if model errors are present. The mathematical expectation of the cost function is given by the expression [4]:

$$E\{V\} = K_{\mathrm{model}} + K_{\mathrm{noise}} \qquad (6)$$

Under the assumption that the noise has a normal distribution, the mean noise contribution to the mathematical expectation of the cost function is given by [7]

$$K_{\mathrm{noise}} = nF - n_p/2 \qquad (7)$$

Hereby, n is the number of outputs, F is the number of frequencies and n_p is the number of unknown parameters, i.e.

$$n_p = \sum_{i=1}^{n}\sum_{j=1}^{n}(z_{[i,j]} + 1) + d \qquad (8)$$

3. EVALUATION OF MODEL EXTRACTION FROM MEASUREMENT DATA

The device under test (DUT) used in this evaluation was a Low-Pass tubular filter with cut-off frequency at 2,2 GHz. The measurements were performed on a HP 8510B network-analyser. Settings were as follows:

Range	45 MHz - 10 GHz
Mode	stepped
Averages	100
Dwell time	20 ms
Nbr points	801

An adapter removal calibration was accomplished [4]. Dwell time was included to ensure that all transients are damped.

To illustrate the performance of the method used, S_{11} and the modelling residual are shown in the figures 1 and 2. Note that no difference can be seen in

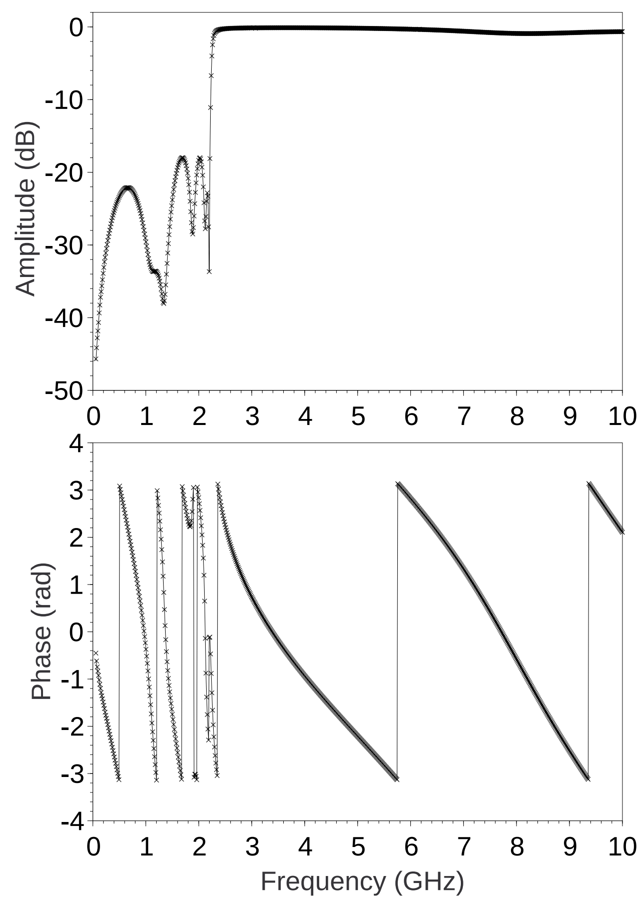

Fig 1: Model (-) and measurements (x) of the amplitude (top) and phase (bottom) of S_{11}.

figure 1 between the measurements and the model. To show the error profile, the magnitude of the complex error $|S_m - S|$ is shown in figure 2. An agreement better than -40 dB is obtained for the whole frequency band. Figure 2 also reveals that the com-

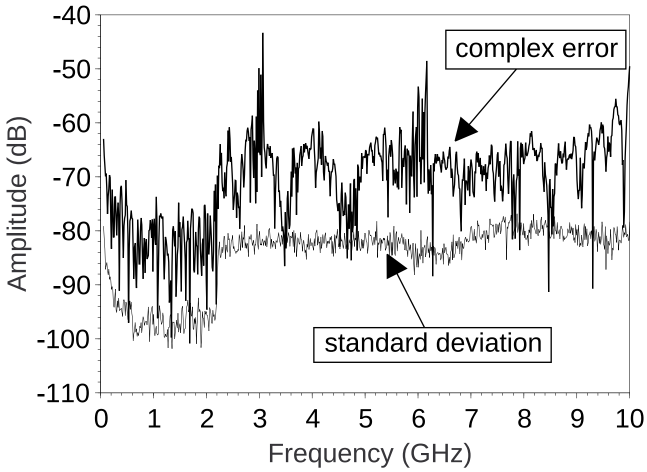

Fig 2: Magnitude of the complex error between model and measurements and standard deviation of the measurements of S_{11}.

plex residual is still larger than the standard deviation on the measurements.

The orders of the nominator and denominator have also to be chosen. To get a first rough idea about the order needed in MIMO, the initial calculations were done in Single Input, Single Output (SISO) because this is much faster. Based on this, the orders of the numerators and denominators were set at 34 (model 34/34). Further MIMO simulations revealed that lowering the model order still gave stable, satisfactory results. Finally, based on the evolution of the cost in function of the order, shown in figure 3, a model of order 24/24 was selected.

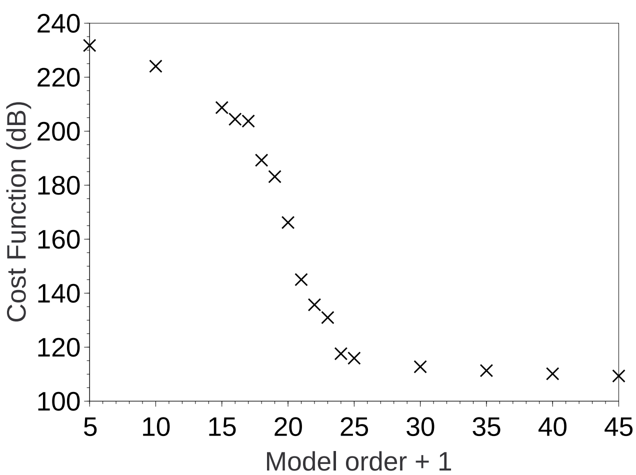

Fig 3: The cost as a function of the model order

It is possible to assign a different order to the numerator of each S-parameter, but here the results obtained with a common numerator order for the different S-parameters were satisfactory.

The selected model has a value of 625376 for the cost function while the noise-only contribution K_{noise} has an expected value of 1540. This discrepancy indicates a contribution of model errors (see K_{model} in equation (6)). It is probable that the peaks at approximately 3 and 6 GHz (figure 2) that appear in the magnitude of the complex error mainly cause this high contribution to the model order. This will be the subject of further research.

Although a black-box approach was used to build the model, the poles are still located on an ellipse in the left half plane, which is the expected behaviour for an inverse Chebeyshev filter. When raising the model order instable poles appear. This is an indication that there is no need for a higher order model, because these poles obviously fit the measurement noise or some small non-idealities (non-linearities). This is supported by figure 4, in which the absolute value of the Function of Dependency [8], and the absolute values of the 95% and 99% fraction bounds are plotted. The FOD is a kind of whiteness test on the complex residual $S_m - S_{estimated}$. The shape of the FOD proves that only small dynamical linear model errors are present. These can be due to the calibration residue.

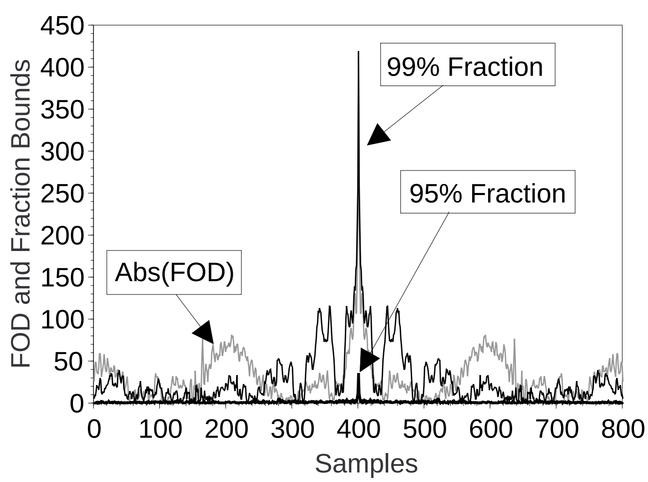

Fig 4: Absolute values of the Function of Dependency (FOD) and the 95% and 99% Fraction Bounds.

4. MODEL EXTRACTION FROM SIMULATION DATA

The goal here is the identification of a model from as few data points as possible. The model built with the proposed techniques allows good interpolation. To prove this two models were built, one starting from a dense set and another from a sparse data set.

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In the shown experiment, 2 data sets describing a multipole filter were generated by means of an EM-field solver (HP-momentum [9]). A dense data set (m1500) describes the S-parameters of the filter in 1500 frequency points while a sparse data set (m122) describes the same filter in only 122 frequency points. To show the model's performance, the results found for S_{22} are plotted. The estimation is done in SISO and the model order is 41/43. An MLE weighting is used.

As shown in the figures 5 and 6, there is only a small loss in performance (<-30 dB over the whole frequency band) caused by the reduction of data points. This indicates that the proposed method can very easily be used to interpolate simulation results obtained form a field solver.

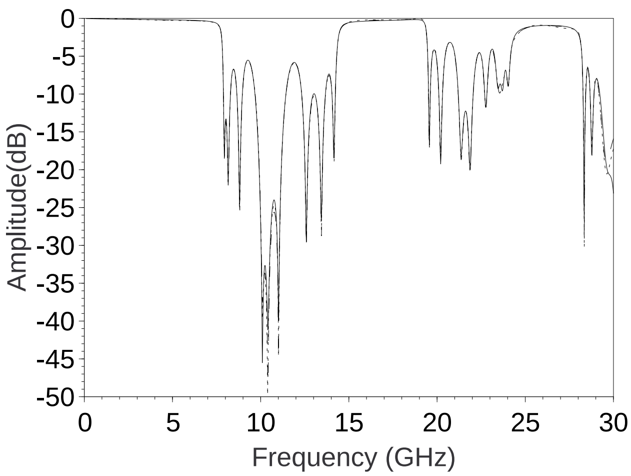

Fig 5: Amplitude of S_{22}: m1500 (__), Model based on m1500 (--), Model based on m122 (...)

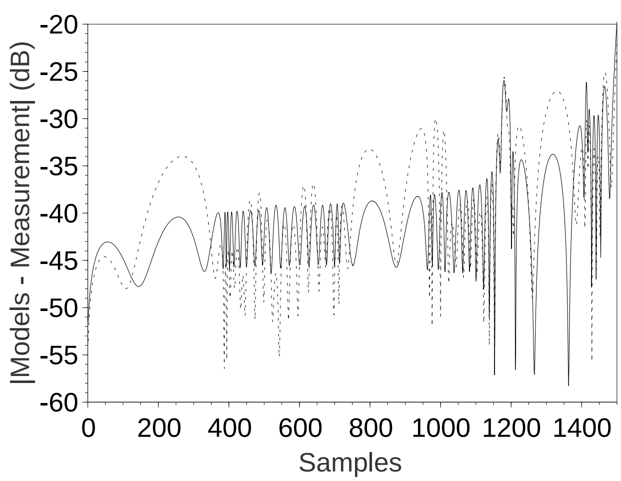

Fig 6: Error between m1500 and models based on the m1500 (__) and m122 (...) data set respectively.

5. CONCLUSIONS

This paper presents a method to build accurate linear models starting from measurements as well as from simulation data. Although a black box approach was used, these models are capable of extracting stable, meaningful poles from standard measured data. On the other hand, the very same algorithms perform extremely well as an interpolator for EM-field simulator outputs.The used techniques are numerically well conditioned, hence high order models can be calculated.

A very important point is that the proposed method allows for model validation and requires only minimal user interaction.

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